Pertanika J. Soc. Sci. & Hum. 33 (3): 1037 - 1058 (2025)

SOCIAL SCIENCES & HUMANITIES

Journal homepage: http://www.pertanika.upm.edu.my/

Review Article

The Effect of Problem-Based Learning, Cooperative Learning, and Discovery Learning on Mathematical Problem-Solving Skill: A Meta-Analysis and Systematic Review

Xueting He*, Ismail Norulhuda and Mengyao He

Faculty of Social Sciences and Humanities, School of Education, Universiti Teknologi Malaysia, 81310, Skudai, Malaysia

ABSTRACT

This meta-analysis aims to systematically assess the effect of Problem-Based Learning, Cooperative Learning, and Discovery Learning models on Mathematical Problem-Solving Skills and determine which strategy had the most significant effect. A systematic search of the literature from 2015 to 2024 was conducted using Scopus, Web of Science, and Google Scholar. Seventeen studies were included in this study. Comprehensive meta-analysis (CMA) software analysis showed that Problem-Based Learning, Cooperative Learning, and Discovery Learning all had a significant positive effect on Mathematical Problem-Solving Skills compared to traditional teaching methods. In contrast, Discovery Learning had the greatest effect, followed by Cooperative Learning and finally Problem-Based Learning. In addition, a heterogeneity analysis revealed that the educational level and sample size were the sources of heterogeneity among the studies. The results of this study provide guidance for selecting effective instructional strategies and provide direction for future research exploring sources of heterogeneity.

Keywords: Cooperative learning, discovery learning, mathematical problem-solving skills, problem-based learning

ARTICLE INFO

Article history:
Received: 07 August 2024
Accepted: 20 December 2024
Published: 26 June 2025

DOI: https://doi.org/10.47836/pjssh.33.3.05

E-mail addresses:
hexueting@graduate.utm.my (Xueting He)
p-norulhuda@utm.my (Ismail Norulhuda)
hemengyao@graduate.utm.my (Mengyao He)
*Corresponding author

INTRODUCTION

Problem-solving is one of the goals of the mathematics curriculum and plays an important role in solving unconventional real-life problems (Orton, 2004). Mathematical problem-solving skills enable students to relate mathematical concepts to everyday life and develop

e-ISSN: 2231-8534

solution strategies, thereby developing high levels of thinking skills (King et al., 2016) and positive attitudes, such as never giving up, and hard work (Pimta et al., 2009). Currently, most countries have made improving students' problem-solving skills an important goal of mathematics education, and problem-solving has become a hot topic in international research on mathematics curricula and teaching (Kolar & Hodnik, 2021; Stacey, 2005).

Although the skill is emphasized all over the world, poor mathematical problem-solving skills still exist. Research has shown that poor mathematical problem-solving skills exist among students in many countries. For example, the United States (Montague & Applegate, 2000), the Philippines (Culaste, 2011), China (Hua, 2019), Indonesia (Mailisman & Ikhsan, 2020), Malaysia (Ozrecberoglu et al., 2022) and Singapore (Hung et al., 2022). Therefore, improving students' mathematical problem-solving skills remains an important issue in education.

The traditional approach to teaching mathematics emphasizes teacher-centered knowledge transfer and mechanical memorization, with less challenging tasks and problems set by the teacher (Hendriana et al., 2018; Pandu & Prabaningrum, 2020). They also argue that this teaching method limits students' deep understanding of concepts, weakens their interest and motivation in learning, and neglects the development of their mathematical problem-solving skills. Because of this, finding more effective teaching strategies to make up for the shortcomings of traditional teaching methods has become a pressing issue.

This study uses constructivist theory as a theoretical foundation. According to constructivist theory, learning is a student-centered process in which students actively construct knowledge through interaction with the environment, exploring solutions, and addressing real-world problems (Vygotsky, 1978). In educational applications, constructivist theory guides student-centered instructional strategies such as problem-based learning (PBL; Darma et al., 2018), cooperative learning (CL; Ridwan & Hadi, 2022), and discovery learning (DL; Tumurun et al., 2016). Research has shown that student-centered instructional strategies have been shown to improve students' mathematical problem-solving skills (Ukobizaba et al., 2021). These instructional strategies aim to improve students' MPSS by stimulating their initiative and creativity in different ways.

PBL is a student-centered, teacher-led instructional model in which students solve problems through problem-posing, independent learning, and group work (Darma et al., 2018). The PBL model creates a learning environment in which problems are used as a starting point and then solved by acquiring new knowledge (Mushlihuddin et al., 2018; Parno et al., 2019). They also argue that following the stages of PBL allows students to be directly involved in investigating a problem and finding a solution so they can take the initiative in solving the problems they face. Several studies have collectively shown that PBL improves students' mathematical problem-solving skills. It has been found that PBL leads to more active learning, self-confidence, creativity and better communication and joint problem-solving (Hendriana et al., 2018; Mulyono & Hadiyanti, 2018). However, some studies still conclude that PBL implementation does not significantly affect students' mathematical problem-solving skills (Panjaitan & Hutauruk, 2019). Therefore, the overall effect of the current PBL on mathematical problem-solving skills needs to be explored.

CL is an instructional model that relies on students working together to better understand a subject (Ridwan & Hadi, 2022). According to Haller et al. (2000), CL focuses on group learning and individual learning. The CL model produces the best learning outcomes when students with different knowledge and skills help each other in group activities. The CL model has been shown to have a positive effect on students' mathematical problem-solving skills (Mukeriyanto et al., 2020; Saputra et al., 2020; Umam et al., 2017). Mukeriyanto et al. (2020) argue that this is because collaborative learning also stimulates students' creativity and latent abilities to help to learn to develop problem-solving thought processes. According to Saputra et al. (2020), collaborative learning promotes active participation in group discussions and communication; collaborative learning makes it easier for students to understand and memorize concepts and apply these mathematical concepts to solve problems.

According to Tumurun et al. (2016), the DL model emphasizes student-centered learning by allowing students to be more active in their learning and search for materials, thus making learning more meaningful than traditional learning models. Studies have shown that DL has a significant positive impact on mathematical problem-solving skills (Pratiwi et al., 2020; Trawikhi et al., 2019). Research has shown that DL improves the quality of learning and cognitive level of students, thus making learning more meaningful (Bajah & Asim, 2002; Martins & Oyebanji, 2000).

Through a systematic literature review and meta-analysis, this study aims to assess the effect of PBL, CL, and DL on mathematical problem-solving skills. The specific research questions are as follows:

1. What are the effects of PBL, CL and DL on mathematical problem-solving skills?
2. What are the differences in the effects of PBL, CL and DL on mathematical problem-solving skills?

To answer these research questions, based on the literature review, the following research hypotheses were formulated for this study:

H1: There is a significant positive effect of PBL on students' mathematical problem-solving skills.

H2: There is a significant positive effect of CL on students' mathematical problem-solving skills.

H3: There is a significant positive effect of DL on students' mathematical problem-solving skills.

H4: There is a significant difference in the effectiveness of PBL, CL and DL in improving mathematical problem-solving skills.

Studies have explored the effects of PBL, CL, and DL on mathematical problem-solving skills separately, as well as meta-analyses of the effects of PBL on MPSS. However, there is a lack of meta-analyses and systematic reviews of CL and DL based on mathematical problem-solving skills, as well as comparisons among the three. This paper retrieved relevant studies between 2015 and 2024 for meta-analysis and systematic review to fill the current research gap. By analyzing and comparing the effects of PBL, CL, and DL on mathematical problem-solving skills, this study clarifies which instructional strategy is more effective in which situation. This informs and supports educators in their choice of instructional strategies.

METHODS

Research Design

Using a systematic literature review and meta-analysis, this study was designed to assess the effects of three instructional strategies, PBL, CL, and DL, on mathematical problem-solving skills. Meta-analysis is a statistical method used to synthesize the results of multiple mean-variance calculations to improve the estimation power of different studies (Lipsey & Wilson, 2001). First, existing studies were searched for and obtained through keywords. Next, inclusion and exclusion criteria were specified. Then, literature was retrieved, coding was performed, and statistics were analyzed and interpreted.

Literature Search Strategy

The researchers explored Web of Science and Scopus using a combination of Boolean operator pairs. The following combinations of Boolean operators were used in the retrieval stage: "problem-solving skill*" AND "math*", "problem-solving abilit*" AND "math*", "problem-solving competenc*" AND 'math*', 'problem-solving capacity' AND 'math*'. To obtain more relevant articles, Google Scholar was searched using the keywords "Cooperative Learning AND Mathematical Problem-Solving Skill", "discover learning AND Mathematical Problem-Solving Skill". To broaden the scope of research related to Cooperative Learning, "group learning" and "team learning", which express the same teaching style as "Cooperative Learning", were searched as keywords.

To ensure the relevance and accuracy of the meta-analysis to the research topic, the following detailed inclusion and exclusion criteria were established in this study:

1. An automated tool was used to limit the period from 2015 to 2024, and the types of articles were limited to articles, conference papers (proceeding papers), and the language was limited to English.

These criteria were done through the automated system tool.

2. Only studies that examined the three instructional strategies of PBL, CL and DL were included; some studies that belonged to the teaching strategies but were not of these three types were excluded. In addition, for these three instructional strategies, this study expanded the keywords. For example, DL and guided DL were included in this study. Because both emphasize students discovering solutions to problems through their active learning (Hosnan, 2014). In this process, the teacher only plays an instructional and guiding role. Group and team-assisted learning were also included in the CL meta-analysis. This is because these strategies emphasize group work. The field of study is going to be the field of mathematics education. Those studies that examined the areas of physics and chemistry were excluded.

3. Studies had to report changes in mathematical problem-solving skills; studies that did not explore changes in this skill or only referred to general problem-solving skills were excluded.

4. Studies must be conducted with students (primary, junior high school, senior high school, and college/university); otherwise, they will be excluded.

5. The results section must report quantitative data and include effect sizes. Studies that had quantitative data but did not specify the effect size needed for this study were excluded.

As shown in Figure 1, the initial search identified 3884 potential studies. The titles

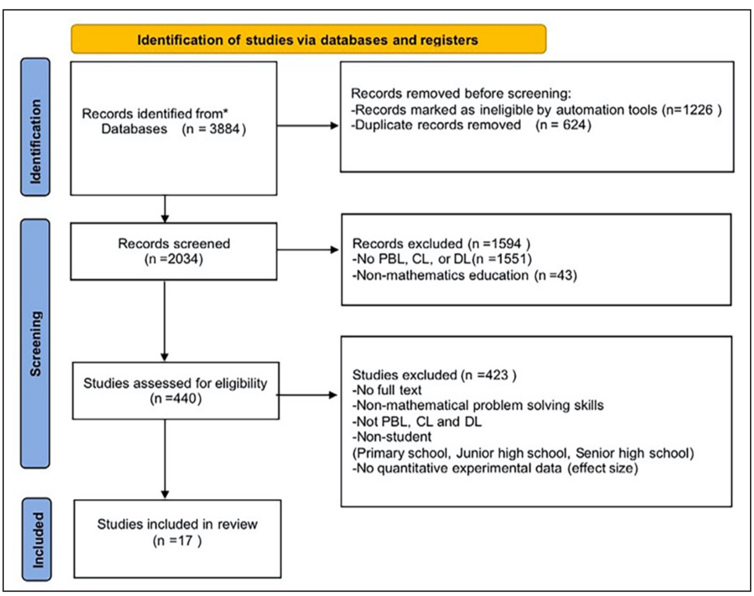

Figure 1. The study selection process

and abstracts of 2034 studies were screened, and after 1226, the system automatically excluded 1226 items, and 624 duplicates were removed. Considering that the studies should fit the theme, 1551 studies on non-teaching strategies (PBL, CL, and DL) and 43 studies in non-mathematics education areas were excluded, leaving 440 studies. Finally, 81 studies without full text, non-math problem-solving skills, non-PBL/CL/DL, non-students (primary school, junior high school, senior high school, and college/university), and no quantitative experimental data were again excluded based on the needs of the topic and research questions of this study, for a total of 423 studies. Seventeen eligible studies were ultimately identified. These rigorous inclusion and exclusion criteria ensured the high relevance of the selected studies and provided a solid foundation for the meta-analysis.

Coding Process

The coding process of this study implemented a strict coding procedure to ensure the validity, accuracy and consistency of the extracted data. First, this study clarified the inclusion and exclusion criteria and developed an appropriate PRISMA flowchart to screen qualified studies. The specific inclusion and exclusion criteria are shown in Figure 1. Then, two independent coders extracted the data using a coding sheet. The coding sheet included authors, year of publication, study design, sample size, level of education, and effect size (mean and standard deviation). In case of any inconsistency in the data extracted by the coders, a discussion was held to determine the criteria for consistency, which resulted in 100% coding consistency. Finally, the data obtained from the coding sheet were entered into an Excel sheet to be organized, saved and meta-analyzed using CMA software.

Data Extraction

This meta-analysis aimed to clarify the effect of the three teaching strategies, PBL, CL and DL, on mathematical problem-solving skills. Therefore, data extracted from each study representative of Mathematical Problem-Solving Skills was used as the result of this investigation. Effect sizes and information to explore sources of heterogeneity were extracted from 17 studies. The specific data extracted from this study are as follows: author's name, Mean, standard deviation, sample size, educational level, experimental design (pre-test, post-test design and post-test only design), and year of publication. Two independent researchers will carry out this process to ensure the accuracy and consistency of the data.

Data Analysis

Data processing for meta-analysis was used to quantify the effect of PBL, CL, and DL on mathematical problem-solving skills. Data that met the criteria were imported into the comprehensive meta-analysis software to calculate the overall effect sizes of PBL, CL, and DL on mathematical problem-solving skills. To ensure the accuracy of the overall effect sizes, the effect model to be used needed to be determined. Therefore, an analysis of heterogeneity between studies is needed. A fixed-effects model is used when the actual effect sizes of all studies are the same (Borenstein et al., 2021). When the actual effect sizes of the studies differed according to factors such as participants' age, education level, or class size, this represented large heterogeneity among the studies; thus, using a random effects model (Üstün & Eryılmaz, 2014).

The selection of fixed-effects and random-effects models is mainly based on the Q statistical test and the statistical value I^2. The Q value assesses heterogeneity among study effect sizes, but only the p-value is needed to determine its presence. If the p-value is lower than 0.05, this indicates a difference between the studies, and the random-effects model is preferred over the fixed-effects model. The statistical value I^2 indicates the extent of the difference between the studies, i.e., the degree of heterogeneity (Hedges & Olkin, 2014). If I^2 is greater than 50%, it indicates a high degree of heterogeneity in the effect sizes of these studies.

Then, publication bias tests were performed, including funnel plots, the classic fail-safe N test, and Egger's regression test. According to Egger et al. (1997), symmetry of the funnel plot indicates no publication bias and vice versa. Because of the subjective nature of observations and judgments in funnel plots, Classic fail-safe N and Egger's regression test validated the conclusions drawn from the funnel plots. Classic fail-safe N value indicates how many additional unpublished and negative studies are needed to make the overall effect size become insignificant. According to Rosenthal (1986). If the classic fail-safe N value is greater than 5k+10 (where k is the number of studies included in the meta-analysis), it indicates that the effect sizes obtained in the meta-analysis are robust and not subject to publication bias. In addition, Egger's test was also used to justify this conclusion. Rosenthal also argued that if the *p*-value in Egger's test is less than 0.05, it indicates that publication bias does not affect the overall effect size.

Lipsey and Wilson (2001) proposed moderation analysis to identify sources of heterogeneity. In this study, subgroup analyses were conducted to explore sources of heterogeneity. It was possible to determine the subgroup on which each of the three instructional strategies had a more significant effect by reading the effect sizes. The effect sizes were interpreted with reference to Thalheimer and Cook's (2002): 0.15 is negligible, 0.4 is small, 0.75 is medium, 1.10 is large, 1.45 is very large, and over 1.45 is huge. This method is performed in conjunction with the Q statistical test, where both Q and p-values are used to assess the effect of factors on the overall effect size. However, only the p-value is needed to confirm heterogeneity. If the p-value is less than 0.05, the factor is a source of heterogeneity and is a factor that influences the overall effect size of the study.

RESULT

This study examines the effect of PBL, CL and DL on mathematical problem-solving

Table 1
Basic information about the studies accepted

	Category	PBL	CL	DL	Total
Number of studies		6	6	5	17
Total sample size (experimental group)		197	193	155	545
Total sample size (control group)		195	187	155	537
Educational level	Primary school	1	1	0	2
	Junior high school	0	3	4	7
	Senior high school	3	1	1	5
	University/College	2	1	0	3
Experimental design	Pre-test & post-test	5	3	3	11
	Only post-test	1	3	2	6
Sample size	11–20	1	1	0	2
	21–30	0	1	2	3
	31–40	4	3	3	10
	41–50	1	1	0	2

skills. Table 1 provides basic information about the studies accepted for the meta-analysis.

According to this study's inclusion and exclusion criteria, 17 studies that met the specified criteria were meta-analyzed. These comprised five studies that examined the effects of DL on mathematical problem-solving skills, six studies that investigated the effects of PBL on MPSS, and six studies that explored the effects of CL on mathematical problem-solving skills. The total sample sizes of the experimental and control groups were 545 and 537, respectively. The studies covered different educational levels, experimental designs, and sample sizes.

Analysis of Overall Effect Size and Heterogeneity of the Studies

The studies on PBL, CL and DL were tested for heterogeneity to determine the effect model used. The overall effect size of these three instructional strategies on mathematical problem-solving skills was also tested according to the effect model used. Table 2 demonstrates the overall effect size and heterogeneity of the PBL, CL and

Table 2
Overall effect sizes and degree of heterogeneity of PBL, CL, and DL studies

Teaching Strategy	Studies Number	Effect Size	95% CI	Null hypothesis test (2-Tail)		Heterogeneity		
				Z	*p*-value	Q-value	*p*-value	I^2
PBL	6	0.690	[0.299;1.082]	3.456	0.001	17.268	0.004	71.045
CL	6	1.145	[0.391;1.899]	2.957	0.003	56.226	0.000	91.107
DL	5	1.427	[0.998;1.856]	6.522	0.000	11.580	0.021	54.458

DL studies. Figures 2, 3, and 4 demonstrate the forest plots of effect sizes for the PBL, CL, and DL studies under the random effects model, respectively.

First, the heterogeneity analysis of the six PBL studies showed Q = 17.268, $p < 0.05$, and $I^2 = 71.045\%$, indicating a high degree of heterogeneity between the effect sizes of these six studies. Therefore, a random effects model was used. The random effects model detected an overall effect size of 0.690 (95% CI: 0.299 to 1.082) for PBL on mathematical problem-solving skills and was a medium effect size (Figure 2). This implies that implementing PBL instructional strategies had a significant positive effect on mathematical problem-solving skills.

Then, the results of the heterogeneity analysis of the six CL studies showed Q = 56.226, $p < 0.05$, and an I^2 value (91.107%) > 50%, indicating that there was a high degree of heterogeneity in the six studies exploring the effect of CL on students' mathematical problem-solving skills. Thus, the overall effect size of CL on mathematical problem-solving skills was 1.145 (95% CI: 0.391 to 1.899), which is a very large effect size, as detected by the random effects model (Figure 3). This means that CL has a

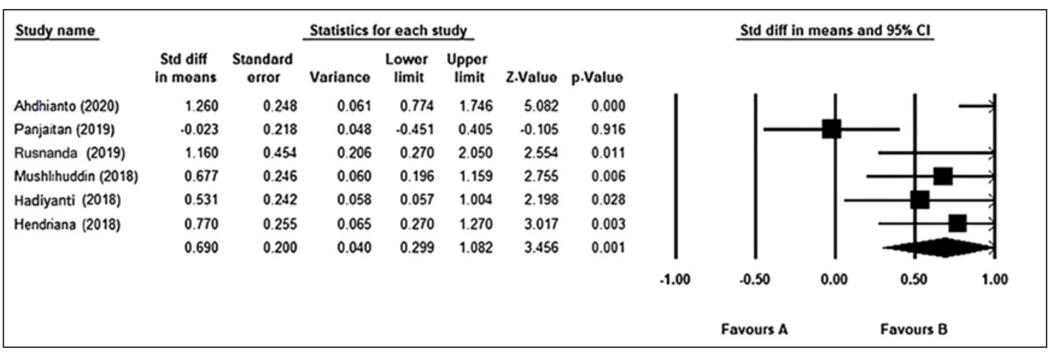

Figure 2. Forest plot of effect sizes of PBL studies under the random effects model
Note. Favors A represents traditional teaching, and Favors B represents PBL

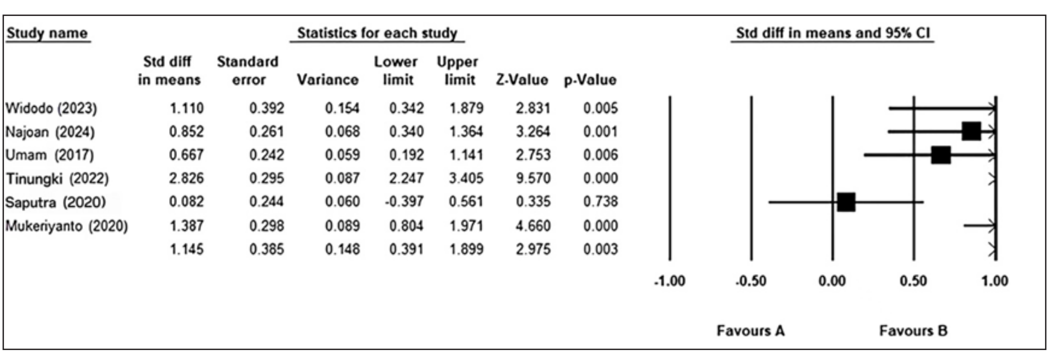

Figure 3. Forest plot of effect sizes for CL studies under random effects modeling
Note. Favors A represents traditional teaching, and Favors B represents CL

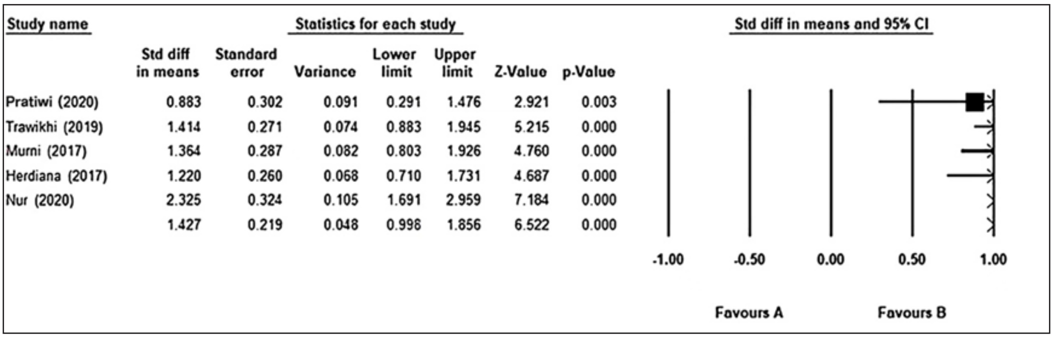

Figure 4. Forest plot of effect sizes for the DL study under random effects modeling
Note. Favors A represents traditional teaching, and Favors B represents DL

significant positive effect on mathematical problem-solving skills.

Finally, the heterogeneity analysis of the five DL studies showed Q = 11.580, p = 0.021 ($p < 0.05$), and $I^2 = 54.458$ ($I^2 > 50\%$), which indicates that there is a high degree of heterogeneity among the effect sizes of these six DL studies. The overall effect size of DL on mathematical problem-solving skills after random effects model testing was 1.427 (95% CI: 0.998 to 1.856), which is a very large effect size (Figure 4). This means that DL has a significant positive effect on mathematical problem-solving skills.

The data show that the overall effect size of DL (d = 1.427) is larger than the overall effect size of CL (d = 1.145), which suggests that DL has a more significant effect on improving students' mathematical problem-solving skills. The heterogeneity of DL ($I^2 = 54.458\%$) is lower than that of CL ($I^2 = 91.107\%$), which suggests that the instructional effects of CL-related studies are more consistent and stable, and the reliability is stronger. In contrast, PBL had the lowest effect size (d = 0.690) and higher heterogeneity (I^2=71.045%). Comparatively, PBL had a smaller overall effect size.

Publication Bias

This meta-analysis included 17 studies. Publication bias detection was performed, including a funnel plot, Classic fail-safe N and Egger's regression test to ensure the accuracy and reliability of the analysis results. If publication bias exists, Duval and Tweedie can use the trim and fill method to adjust.

First, a PBL, CL, and DL meta-analysis revealed that the three funnel plots produced were generally symmetrically distributed (Figures 5, 6, and 7). To validate this result, Classic fail-safe N was used to estimate how many studies with unpublished or negative results would be needed to make the overall effect size nonsignificant. The analysis showed that the Classic fail-safe N numbers for PBL, CL, and DL were 57, 137, and 155, respectively. These numbers were all greater than 5k+10 (where k is the number of studies included in the meta-analysis), indicating robust results. This suggests that the overall effect sizes of all three instructional strategies, PBL, CL, and DL, are unlikely to be influenced by unpublished or negative research.

In addition, this conclusion is further supported by the results of Egger's regression test, which for PBL, CL and DL-related studies were $p = 0.32775 > 0.05$, $p = 0.35014 > 0.05$, and $p = 0.36466 > 0.05$. This suggests a low likelihood of publication bias. The results of the funnel plot, Classic fail-safe N, and Egger's regression test collectively indicate that no publication

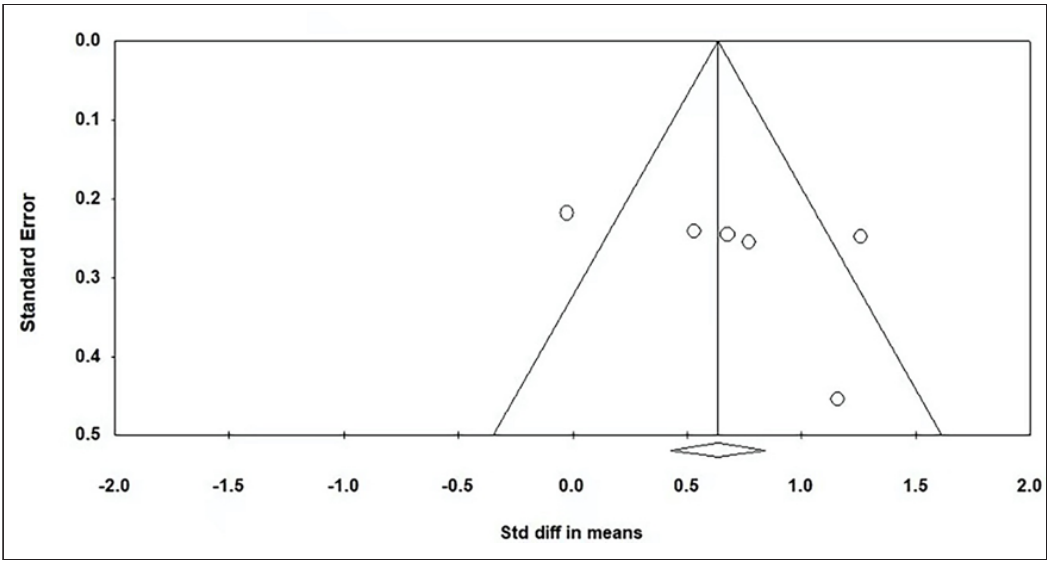

Figure 5. Funnel plot of PBL study

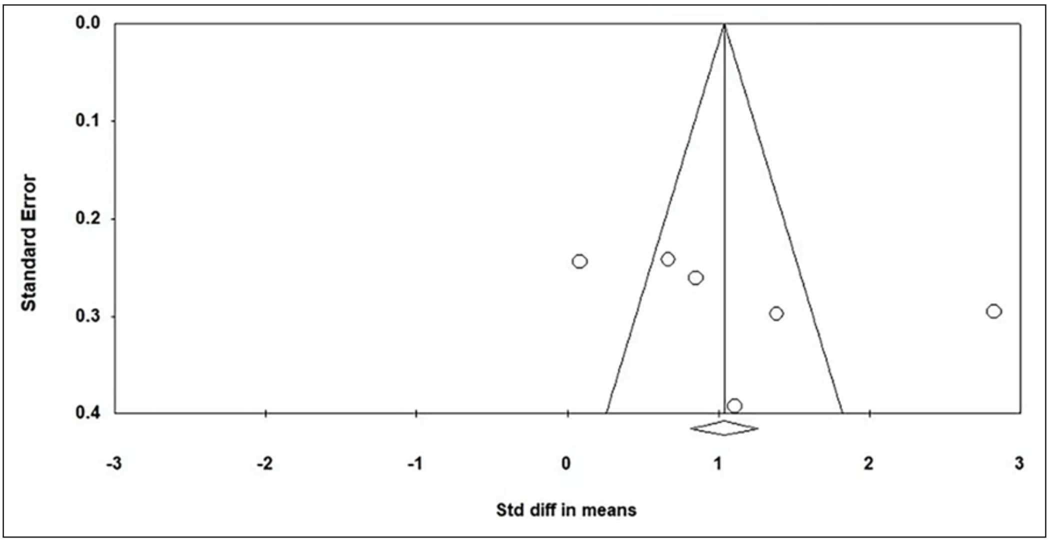

Figure 6. Funnel plot of CL study

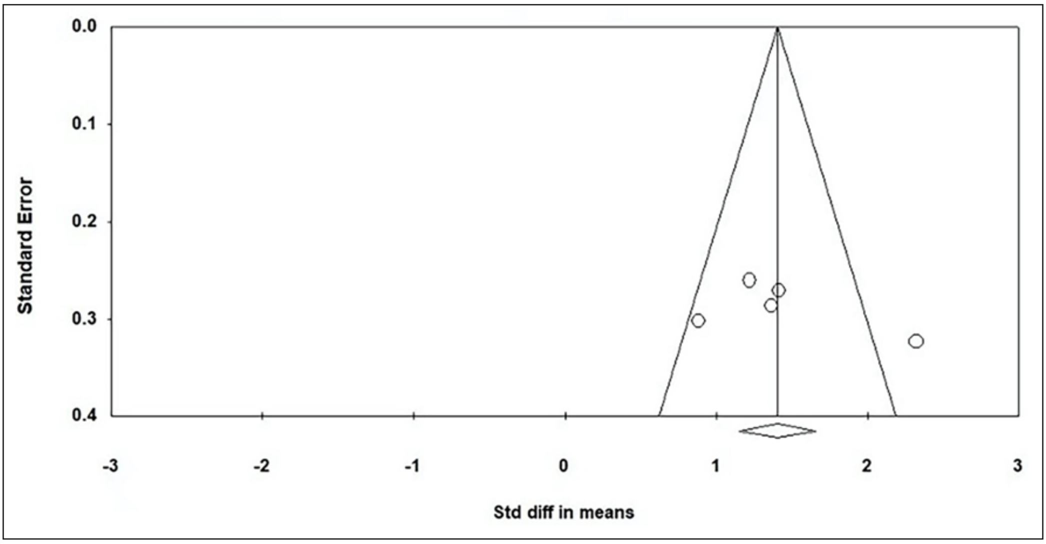

Figure 7. Funnel plot of DL study

bias in the meta-analysis seriously threatens the effect sizes of the three instructional strategies, PBL, CL, and DL. Therefore, the credibility of these findings is high.

Moderation Analysis

Since there was significant heterogeneity among the overall effect sizes of the six PBL, five CL, and six DL studies, it was necessary to perform a moderation analysis to explore sources of heterogeneity. Subgroup analyses were performed in this study. Table 3 demonstrates the results of overall effect sizes and heterogeneity by educational level, experimental design, and sample size.

In terms of education level, significant differences in effect sizes between education levels were found in the studies of PBL (Q = 6.040, $p = 0.049 < 0.05$) and CL (Q = 52.435, $p = 0.000 < 0.05$). No significant difference in effect size between education levels was found in the study of DL (Q = 3.111, $p = 0.078 > 0.05$). This suggests that education level was a source of heterogeneity between PBL and CL studies but not between DL studies.

In addition, the effect sizes of PBL were 1.260, 0.412, and 0.787 for primary school, senior high school, and college, respectively. This suggests that the effect of PBL was significant at the primary and college levels, but not at the senior high school level.

For CL, the effect sizes were 0.852 (primary school), 1.020 (junior high school), 0.082 (senior high school), and 2.826 (college), indicating significant effects at the primary, junior high, and college levels but not at the senior high level. Finally, DL showed significant effects at the junior and senior high school levels, with effect sizes of 1.555 (junior high school) and 0.883 (senior high school). In summary, it shows that PBL is more suitable for primary school and college level, CL is more suitable for college, junior high school, and primary

Table 3
Overall effect size and heterogeneity for education level, experimental design, and sample size

		Studies Number	Effect Size	Null Hypothesis Test (2-Tail)		Heterogeneity	
				Z-value	p-value	Q-value	P-value
Educational Level							
PBL	Primary school	1	1.260	5.082	0.000	6.040	0.049
	Senior high school	3	0.412	1.713	0.087		
	University/College	2	0.787	3.639	0.000		
	Overall	6	0.809	5.997	0.000		
CL	Junior high school	3	1.020	4.302	0.000	52.435	0.000
	primary school	1	0.852	3.264	0.001		
	Senior high school	1	0.082	0.335	0.738		
	University/College	1	2.826	9.570	0.000		
	Overall	6	1.062	8.271	0.000		
DL	Junior high school	4	1.555	6.919	0.000	3.111	0.078
	Senior high school	1	0.883	2.921	0.003		
	Overall	5	1.307	7.111	0.000		
Experimental Design							
PBL	Post	1	0.531	2.198	0.028	0.340	0.560
	Pre-post	5	0.733	2.956	0.003		
	Overall	6	0.629	3.637	0.000		
CL	Post	3	1.020	4.302	0.000	0.076	0.783
	Pre-post	3	1.246	1.585	0.113		
	Overall	6	1.039	4.576	0.000		
DL	Post	2	1.134	4.717	0.000	1.534	0.215
	Pre-post	3	1.628	5.103	0.000		
	Overall	5	1.313	6.838	0.000		
Sample Size							
PBL	11–20	1	1.160	2.554	0.011	11.295	0.004
	31–40	4	0.807	5.081	0.000		
	41–50	1	-0.023	-0.105	0.916		
	Overall	6	0.567	4.589	0.000		
CL	11–20	1	1.110	2.831	0.005	37.*925*	0.000
	21–30	1	1.387	4.660	0.000		
	31–40	3	0.528	2,279	0.023		
	41–50	1	2.826	9.570	0.000		
	Overall	6	1.360	6.688	0.000		
DL	21–30	2	1.134	4.717	0.000	1.534	0.215
	31–40	3	1.628	5.103	0.0000		
	Overall	5	1.313	6.838	0.000		

school, and DL is more suitable for junior high school and senior high school.

Experimental design is considered one of the factors affecting inter-study heterogeneity. This study categorized the experimental design into pre-test, post-test, and post-test-only designs. The results showed that the effect sizes of PBL (Q = 0.340, $p = 0.560$), CL (Q = 0.076, $p = 0.783$), and DL (Q = 1.534, $p = 0.215$) did not differ significantly under either classification. This implies that neither pre-test post-test nor post-test only design in the experimental design contributes to the heterogeneity in the effect sizes of students' mathematical problem-solving skills when implementing these instructional strategies of PBL, CL, and DL.

However, depending on the research design, the effect sizes of PBL, CL, and DL differed in pre-test-post-test design and post-test-only design. In PBL, CL, and DL, the effect sizes were 0.733, 1.246, and 1.628 for pre-test post-test design, and 0.531, 1.020, and 1.134 for post-test only design, respectively. A comparison of the effect sizes under the different experimental designs revealed that the effect sizes of all three instructional strategies in the pre-test-post-test design are greater than those of the post-test only design. This means that the pre-test-post-test design is better at revealing the effectiveness of instructional strategies.

The results of another subgroup analysis indicated that sample size significantly influenced the effect sizes of the three instructional methods: PBL, CL, and DL. The results of the heterogeneity analyses for PBL, CL, and DL indicated that sample size was not a source of heterogeneity in the between-study effect sizes for DL (Q = 1.534, $p = 0.215$) but rather a source of heterogeneity for PBL (Q = 11.295, $p = 0.004$) and CL (Q = 37.925, $p = 0.000$).

Based on the PBL study, the effect values were higher in the groups with sample sizes of 11-20 (d = 1.160) and 31-40 (d = 0.807). The effect value in the group with a sample size of 41-50 (d = -0.023) was close to zero or even negative. This suggests that implementation is more effective when the number of PBL students in the class is small. The effect sizes of CL in groups with 41–50, 21–30, 11–20, and 31–40 are 2.826, 1.387, 1.110, and 0.528, respectively. This suggests that CL is more appropriate when the number of students is large. The effect sizes of DL in the groups with 21–30 and 31–40 are 1.134 and 1.628. This suggests that DL is very useful in experiments with 21–30 and 31–40 sample sizes.

Sensitivity Analysis

Because of the high level of heterogeneity, sensitivity analysis was used to assess the reliability of the results (Table 4). Sensitivity analysis requires analyzing the overall effect of the remaining studies after excluding each study individually. This meta-analysis performed this operation separately for PBL, CL, and DL. The overall effect size for PBL was 0.690, z = 3.456, $p = 0.001$ ($p < 0.05$), for CL, it was 1.145, $z = 2.975$, $p = 0.003$ ($p < 0.05$); and for DL, it was 1.405, $z = 10.972$, $p = 0.000$ ($p < 0.05$). The results

Table 4
Sensitivity statistics for PBL, CL and DL under the random effects model

Teaching Strategy	Effect Size	95% CI	*Z*-value	*p*-value
PBL	0.690	[0.229,1.082]	3.456	0.001
CL	1.145	[0.391,1.899]	2.975	0.003
DL	1.405	[1.154,1.656]	10.972	0.000

of the study show that PBL, CL and DL still have a significant effect on students' mathematical problem-solving skills after excluding different studies.

DISCUSSION

The significant positive effect of PBL on mathematical problem-solving skills is consistent with the findings of previous studies. Students who received PBL instruction scored higher on mathematical problem-solving skills than those who were taught using traditional instructional models (Mulyono & Hadiyanti, 2018; Rusnanda, 2019). This may be because teachers create student-centered learning environments when performing PBL; students can analyze, synthesize, and evaluate these problems and ultimately develop problem-solving skills by asking questions and relating them to real life in group learning (Ahdhianto et al., 2020). Mushlihuddin et al. (2018) argued that PBL allows students to be directly involved in investigating problems and finding solutions and that, ultimately, students are able to solve the problems they face on their own. Hendriana et al. (2018) argued that PBL facilitates student learning, development of creativity and self-information, which leads to communication and joint problem-solving. However, Panjaitan and Hutauruk (2019) came to the opposite conclusion when they argued that PBL is not better than the conventional learning mode in developing students' Mathematical Problem-Solving Skills. This is because the PBL in that study incorporated a metacognitive approach, which resulted in both PBL and conventional teaching models emphasizing direct teacher instruction.

The significant positive effect of CL on mathematical problem-solving skills is consistent with the findings of several studies. That is, the CL model has a positive effect on students' mathematical problem-solving skills (Tinungki et al., 2022; Umam et al., 2017; Widodo et al., 2023). This is because CL can encourage students to learn more actively through group learning, stimulate students' creativity and potential, and help students develop problem-solving thinking (Mukeriyanto et al., 2020; Najoan et al., 2024; Saputra et al., 2020). According to Saputra et al. (2020), CL promotes active participation in group discussions and communication; CL makes it easier for students to understand and memorize concepts and apply these mathematical concepts to solve problems.

The significant positive effect of DL on mathematical problem-solving skills is consistent with the findings of previous studies that students who embraced the DL model had better mathematical problem-solving skills than those who followed the traditional learning model (Pratiwi et al., 2020; Trawikhi et al., 2019). Murni et al. (2017) argued that the difference in mathematical problem-solving ability between students adopting the DL model and those adopting traditional learning is caused by the learning process. Specifically, during the learning process, students can focus their attention and increase their interest as they are exposed to the real world; by actively participating in group discussions, they can identify and determine strategies for solving problems. Nur et al. (2020) argued that during the learning process, students can exchange ideas in group interactions and proactively create solutions to math problems. In addition, Herdiana et al. (2017) argued that the DL model, in its application, fosters active learning through discovering oneself and investigating oneself, which is conducive to improving students' ability to understand mathematical concepts.

In terms of the overall effect of the three instructional strategies, DL performs better in enhancing students' mathematical problem-solving skills, followed by CL and PBL. This confirms the findings of Hulukati et al. (2018), who concluded that the guided DL model is more effective than the CL model in enhancing mathematical problem-solving skills. However, the scholar concluded that students' initial mathematical ability has a significant effect on mathematical problem-solving ability. Future research could increase the number of studies to further explore this result and analyze in depth why this result occurred.

CONCLUSION

This study executed a meta-analysis and systematic review to explore the effects of PBL, CL and DL on mathematical problem-solving skills. Table 5 demonstrates the effect sizes of the different teaching strategies (PBL, CL, and DL) as well as their applicable educational level, sample sizes, and experimental design.

The results of the study verified the previously proposed hypotheses (H1, H2, H3, and H4). They showed that PBL, CL, and DL had a significant positive effect on students' mathematical problem-solving

Table 5
Summary of teaching strategies, effect sizes, and contextual variables

Teaching Strategy	Effect Size	Educational Level	Sample Size	Experimental Design
PBL	Medium effect size (0.690)	Primary school University /College	Small classes (11–20)	Pre-test Post-test Design
CL	Large effect size (1.145)	University /College Junior high school Primary school	Large classes (41–50)	Pre-test Post-test Design
DL	Large effect size (1.427)	Junior high school Senior high school	Moderate classes (21–30, 31–40)	Pre-test Post-test Design

skills compared to traditional teaching methods and that there is a significant difference in the effectiveness of these three teaching strategies in improving mathematical problem-solving skills. Specifically, DL had the greatest effect on students' mathematical problem-solving skills, followed by CL and finally PBL. As far as the theoretical perspective is concerned, the results of this study demonstrate the effectiveness of student-centered constructivist theory in developing mathematical problem-solving skills. PBL, CL and DL allow students to engage in collaborative group learning in the real world by creating a student-centered teaching and learning environment. This deepens students' understanding of concepts and stimulates their creativity and potential to develop mathematical problem-solving skills. This is highly consistent with the constructivist theory's emphasis on the active construction of knowledge through student-environment interaction.

In addition, subgroup analyses were conducted, which revealed that educational level and sample size were the sources of inter-study heterogeneity. In terms of education level, PBL was more effective at the primary and college/university levels; CL was more effective at the college/university, junior high school, and primary levels; and DL was more appropriate for the junior high school and senior high school levels. In terms of sample size, PBL was more effective with small class sizes (e.g., 11–20 students); CL was more effective with large class sizes (e.g., 41–50 students); and DL was significant in classes with moderate sizes (e.g., 21–30, 31–40). The study suggests that experimental design is not a source of heterogeneity for PBL, CL, and DL. Notably, the effect sizes of all three instructional strategies were larger in the pre-test post-test design than in the post-test only design. This implies that pre-test-post-test designs are better at revealing the effectiveness of instructional strategies.

Implications of the Study

This study fills the current research gap by comparing three teaching strategies (PBL, CL, and DL). The results of this study have not only theoretical but also practical implications. Through a systematic literature review and meta-analysis, this study confirmed the effectiveness of constructivism theory in guiding instructional strategies. Based on constructivist theory, PBL, CL, and DL all have a positive effect on students' mathematical problem-solving skills. This further supports the importance of constructivist theory in applying teaching strategies. This suggests that a student-centered teaching and learning environment encourages students to engage in collaborative group activities to stimulate their interest and motivation to improve their mathematical problem-solving skills.

In terms of teaching practice, teachers can optimize teaching and learning by choosing relatively appropriate teaching strategies based on the findings of this study, class size and grade level. For example, CL is suitable for large-sized classes (41–50), DL is suitable for medium-sized classes

(21–40), and PBL is suitable for small-sized classes (11–20). The findings of this study will also help future researchers to understand the extent to which these three teaching strategies affect mathematical problem-solving skills and provide theoretical foundations and empirical support for subsequent studies.

Limitations and Recommendations for Future Studies

Due to limited data sources, the number of studies ultimately included in the meta-analysis is small, which may affect the representativeness of the results. Future studies could search more databases (e.g., ERIC, JSTOR, ProQuest) for additional data support. In addition, this study focused on studies from 2015 to 2024, so the number of studies included in the meta-analysis is limited. Future studies could expand the time frame to obtain more data support.

Second, heterogeneity was present in the PBL, CL, and DL studies, and these may have affected the stability of the effect sizes. The sources of heterogeneity explored in this study were limited. Future research could explore sources of heterogeneity by looking at more perspectives. For example, publication year, sample characteristics, and peer review. Specifically, future research groups publications into 2- or 5-year time periods. This would allow for analyzing differences in the effect sizes of these 3 teaching strategies across time periods. Future research could also divide the sample into different cultural and regional contexts (e.g., European, American, Asian, and African regions). This could help find the most appropriate cultural and regional contexts for each teaching strategy. Future research could also analyze peer-reviewed literature against non-peer-reviewed literature. This could identify more reliable studies so that these studies can be used to inform teaching practice.

In response to the sources of heterogeneity in this study (year of publication, sample characteristics, and peer review), future research could compare the categorization of the sources of heterogeneity in more depth. For example, differences in effect sizes for a particular teaching strategy across years of publication or sample characteristics could be analyzed. Then, comparing the classification of these sources of heterogeneity and analyzing the details and context of the teaching strategies in these studies could optimize the application of teaching strategies in practice.

ACKNOWLEDGEMENT

The authors express sincere gratitude to those who contributed valuable insights and assistance throughout this meta-analysis and systematic review.

REFERENCES

Ahdhianto, E., Marsigit, H., & Nurfauzi, Y. (2020). Improving fifth-grade students' mathematical problem-solving and critical thinking skills using problem-based learning. *Universal Journal of Educational Research*, *8*(5), 2012-2021. https://doi.org/10.13189/ujer.2020.080539

Bajah, S., & Asim, A. (2002). Construction and science learning experimental evidence in a nigerian setting. *World Council for Curriculum and Instruction (WCCI) Nigeria*, *3*(1), 105-114.

Borenstein, M., Hedges, L. V., Higgins, J. P., & Rothstein, H. R. (2021). *Introduction to meta-analysis*. John Wiley & Sons.

Culaste, I. C. (2011). Cognitive skills of mathematical problem solving of grade 6 children. *International Journal of Innovative Interdisciplinary Research*, *1*(1), 120-125.

Darma, I. K., Candiasa, I., Sadia, I. W., & Dantes, N. (2018). The effect of problem based learning model and authentic assessment on mathematical problem solving ability by using numeric ability as the covariable. *Journal of Physics: Conference Series, 1040*, Article 012035. https://doi.org/10.1088/1742-6596/1040/1/012035

Egger, M., Smith, G. D., Schneider, M., & Minder, C. (1997). Bias in meta-analysis detected by a simple, graphical test. *BMJ*, *315*(7109), 629-634. https://doi.org/10.1136/bmj.315.7109.629

Haller, C. R., Gallagher, V. J., Weldon, T. L., & Felder, R. M. (2000). Dynamics of peer education in cooperative learning workgroups. *Journal of Engineering Education*, *89*(3), 285-293. https://doi.org/10.1002/j.2168-9830.2000.tb00527.x

Hedges, L. V., & Olkin, I. (2014). *Statistical methods for meta-analysis*. Academic Press.

Hendriana, H., Johanto, T., & Sumarmo, U. (2018). The role of problem-based learning to improve students' mathematical problem-solving ability and self-confidence. *Journal on Mathematics Education*, *9*(2), 291-300.

Herdiana, Y., Wahyudin, W., & Sispiyati, R. (2017). Effectiveness of discovery learning model on mathematical problem solving. *AIP Conference Proceedings, 1868*(1), Article 050028. https://doi.org/10.1063/1.4995155

Hosnan, M. (2014). *Pendekatan saintifik dan kontekstual dalam pembelajaran Abad 21: Kunci sukses implementasi kurikulum 2013* [The scientific and contextual approach in 21st century learning: The key to successful implementation of the 2013 curriculum]. Ghalia Indonesia.

Hua, X. (2019). *Research on the strategy of improving junior high school students' problem-solving skills under the core literacy of mathematics* [Master's thesis, University of Jinan]. https://doi.org/10.27166/d.cnki.gsdcc.2019.000137

Hulukati, E., Zakiyah, S., & Rustam, A. (2018). The effect of guided discovery learning model with superitem test on students' problem-solving ability in mathematics. *Journal of Social Science Studies*, *5*(2), Article 210. https://doi.org/10.5296/jsss.v5i2.13406

Hung, D., Hoe, L. N., Lee, J., Shing, L. S., Yang, W. Z., Mei, L., & Seng, K. T. (2022). Addressing the skills gap: What schools can do to cultivate innovation and problem solving. In D. Hung, L. Wu, & D. Kwek (Eds.), *Diversifying schools. Education in the Asia-Pacific Region: Issues, concerns and prospects* (Vol. 61; pp. 177-192). Springer. https://doi.org/10.1007/978-981-16-6034-4_10

King, F., Goodson, L., & Rohani, F. (2016). *Higher order thinking skills: Definition, teaching strategies, & assessment.* Educational Services Program (Center for Advancement of Learning and Assessment).

Kolar, V. M., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. *European Journal of Educational Research*, *10*(1), 467-483. https://doi.org/10.12973/eu-jer.10.1.467

Lipsey, M. W., & Wilson, D. B. (2001). *Practical meta-analysis*. SAGE Publications.

Mailisman, N., & Ikhsan, M. (2020). Mathematics problem-solving skills of vocational high school students related to the 21st-century education. *Journal of Physics: Conference Series, 1460*(1), Article 012014. https://doi.org/10.1088/1742-6596/1460/1/012014

Martins, O., & Oyebanji, R. (2000). The effects of inquiry and lecture teaching approaches on the cognitive achievement of integrated science students. *Journal of Science Teachers' Association of Nigeria*, *35*(1&2), 25-30.

Montague, M., & Applegate, B. (2000). Middle school students' perceptions, persistence, and performance in mathematical problem solving. *Learning Disability Quarterly*, *23*(3), 215-227. https://doi.org/10.2307/1511165

Mukeriyanto, M., Mastur, Z., & Mulyono, M. (2020). The problem solving ability of students in the cooperative learning simulation of Kancing Gemerincing technique with ethnomathematics nuances reviewed by the cognitive mode. *Unnes Journal of Mathematics Education Research*, *9*(2), 179-184.

Mulyono & Hadiyanti, R. (2018). Analysis of mathematical problem-solving ability based on metacognition on problem-based learning. *Journal of Physics: Conference Series, 983,* Article 012157. https://doi.org/10.1088/1742-6596/983/1/012157

Murni, V., Sariyasa, S., & Ardana, I. (2017). GeoGebra assist discovery learning model for problem solving ability and attitude toward mathematics. *Journal of Physics: Conference Series, 895*(1), Article 012049. https://doi.org/10.1088/1742-6596/895/1/012049

Mushlihuddin, R., Nurafifah, & Irvan. (2018). The effectiveness of problem-based learning on students' problem solving ability in vector analysis course. *Journal of Physics: Conference Series, 948,* Article 012028. https://doi.org/10.1088/1742-6596/948/1/012028

Najoan, R. A. O., Kumolontang, D. F., Ester, K., Tarusu, D. T., Setiawan, B., & Iasha, V. (2024). Impact of the team-assisted individualization learning model on mathematical problem-solving, communication, and self-regulated learning. *International Journal of Innovative Research and Scientific Studies*, *7*(2), 409-419. https://doi.org/10.53894/ijirss.v7i2.2645

Nur, F., Tayeb, T., Widayanti, V. M., Sriyanti, A., & Nurhidayah, N. (2020). Effectiveness of discovery learning model on students' mathematical problem solving ability. *Jurnal Matematika dan Pembelajaran*, *8*(2), 125-139. https://doi.org/10.24252/mapan.2020v8n1a1

Orton, A. (2004). *Learning mathematics: Issues, theory and classroom practice*. Bloomsbury Publishing.

Ozrecberoglu, N., Aydın, S., & Aydın, O. (2022). Students' skills in solving non-routine mathematical problems. *Education Quarterly Reviews*, *5*(2), 446-461. https://doi.org/10.31014/aior.1993.05.02.504

Pandu, Y., & Prabaningrum, C. (2020). Analysing the problem-solving ability through the Problem-Based Learning model on the subject statistics in the grade VIIIB of Kanisius junior high school Kalasan. *Journal of Physics: Conference Series, 1470*, Article 012058. https://doi.org/10.1088/1742-6596/1470/1/012058

Panjaitan, S. M., & Hutauruk, A. J. (2019). Students problem-solving abilities using the problem-based learning model with a metacognitive approach. *Journal of Physics: Conference Series, 1315*, Article 012009. https://doi.org/10.1088/1742-6596/1315/1/012009

Parno, Yuliati, L., & Ni'Mah, B. (2019). The influence of PBL-STEM on students' problem-solving skills in the topic of optical instruments. *Journal of Physics: Conference Series, 1171*, Article 012013. https://doi.org/10.1088/1742-6596/1171/1/012013

Pimta, S., Tayraukham, S., & Nuangchalerm, P. (2009). Factors influencing mathematic problem-solving ability of sixth grade students. *Journal of Social Sciences, 5*(4), 381-385. https://doi.org/10.3844/jssp.2009.381.385

Pratiwi, K. M., Sudiarta, I. G. P., & Suweken, G. (2020). The effect of guided discovery learning model assisted by open-ended student worksheets towards mathematical problem solving ability reviewed of student's emotional intelligence. *Journal of Physics: Conference Series, 1503*, Article 012014. https://doi.org/10.1088/1742-6596/1503/1/012014

Ridwan, M. R., & Hadi, S. (2022). A meta-analysis study on the effectiveness of a cooperative learning model on vocational high school students' mathematics learning outcomes. *Participatory Educational Research*, *9*(4), 396-421. https://doi.org/10.17275/per.22.97.9.4

Rosenthal, R. (1986). Meta-analytic procedures for social science research Sage Publications: Beverly Hills, 1984, 148 pp. *Educational Researcher*, *15*(8), 18-20. https://doi.org/10.3102/0013189X015008018

Rusnanda, R. (2019). Improvement of mathematical problem solving ability through problem based learning model in applied mathematics course II in industrial engineering. *IOP Conference Series: Materials Science and Engineering, 506,* Article 012025. https://doi.org/10.1088/1757-899x/506/1/012025

Saputra, P. R. A., Sariyasa, I., & Suparta, I. N. (2020). MURDER cooperative learning model through edmodo toward mathematics problem solving and motivation of students. *Journal on Emerging Mathematics Education*, *4*(1), 43-52. https://doi.org/10.12928/ijeme.v4i1.15845

Stacey, K. (2005). The place of problem solving in contemporary mathematics curriculum documents. *The Journal of Mathematical Behavior*, *24*(3-4), 341-350. https://doi.org/10.1016/j.jmathb.2005.09.004

Thalheimer, W., & Cook, S. (2002). How to calculate effect sizes from published research: A simplified methodology. *Work-Learning Research*, *1*(9), 1-9.

Tinungki, G. M., Nurwahyu, B., Hartono, A. B., & Hartono, P. G. (2022). Team-assisted individualization type of the cooperative learning model for improving mathematical problem solving, communication, and self-proficiency: Evidence from operations research teaching. *Education Sciences*, *12*(11), Article 825. https://doi.org/10.3390/educsci12110825

Trawikhi, A., Prihandoko, A., Prihandoko, A. C., & Utomo, B. (2019). Development of mathematical learning tools through discovery learning based on lesson study for learning community and their influence with students' problem solving. *Journal of Physics: Conference Series, 1211*, Article 012082. https://doi.org/10.1088/1742-6596/1211/1/012082

Tumurun, S. W., Gusrayani, D., & Jayadinata, A. K. (2016). Pengaruh model pembelajaran discovery learning terhadap keterampilan berpikir kreatif siswa pada materi sifat-sifat cahaya. *Jurnal Pena Ilmiah*, *1*(1).

Ukobizaba, F., Nizeyimana, G., & Mukuka, A. (2021). Assessment strategies for enhancing students' mathematical problem-solving skills: A review of literature. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(3). https://doi.org/10.29333/ejmste/9728

Umam, K., Suswandari, N. A., Wibowo, I. T., & Rohim, S. (2017). The effect of think-pair-share cooperative learning model assisted with ICT on mathematical problem solving ability among junior high school students. In *Workshop Proceedings of the 25th International Conference on Computers in Education* (pp. 94-98). Asia-Pacific Society for Computers in Education.

Üstün, U., & Eryılmaz, A. (2014). Etkili araştırma sentezleri yapabilmek için bir araştırma yöntemi: Meta-analiz [A research method for making effective research syntheses: Meta-analysis]. *Eğitim ve Bilim*, *39*(174). https://doi.org/10.15390/EB.2014.3379

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Widodo, S. A., Turmudi, T., Dahlan, J. A., Watcharapunyawong, S., Robiasih, H., & Mustadin, M. (2023). The sociograph: Friendship-based group learning in the mathematics class. *Infinity Journal*, *12*(1), 27-40. https://doi.org/10.22460/infinity.v12i1.p27-40